Frequencies in Forgetful Timed Automata

Amélie Stainer

Université de Rennes 1, France

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Motivations

Timed Automata

- finite automata enriched with clocks
- usual semantics: Büchi semantics

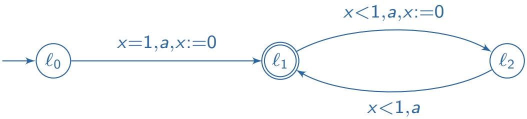
Goal: capturing quantitative behaviors to constrain the usual semantics

Frequency of a run: proportion of time elapsed in critical locations

- Semantics with positive frequency
  - taking into account quantitative aspects
  - Emptiness and Universality problems
- Set of frequencies in a timed automaton

Outline

Timed automata: usual semantics



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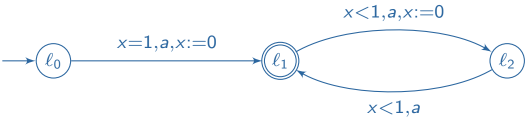

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$(1, a)(1.5, a)(1.75, a)(2.25, a)(2.5, a)\cdots$

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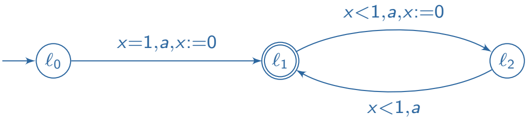

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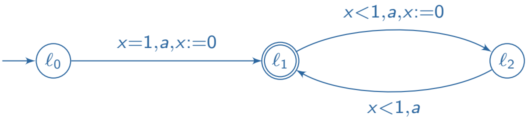

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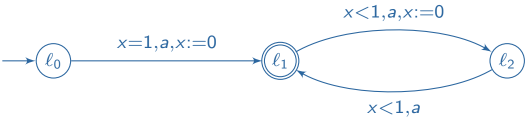

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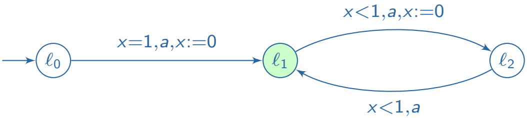
Accepted timed word: timed word with an accepted run

Frequency-based semantics



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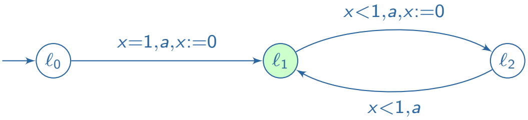

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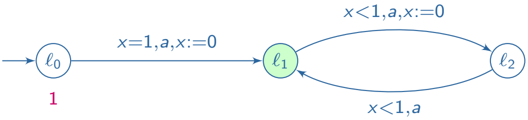

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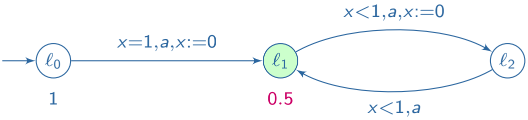

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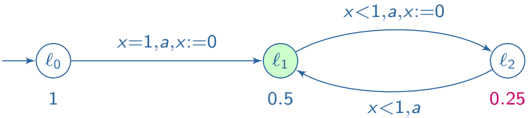

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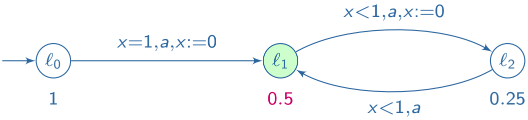

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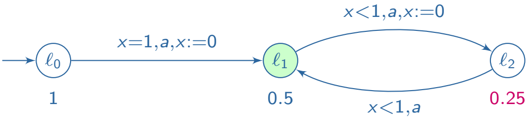

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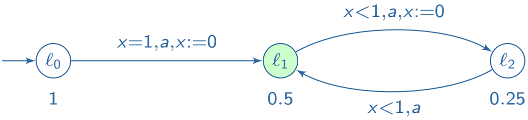

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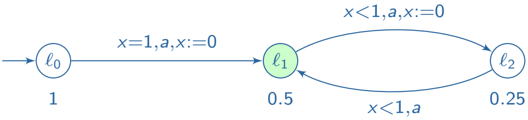

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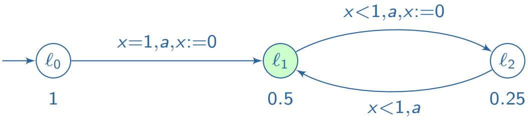

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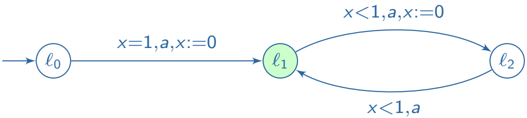
Semantics with positive frequency (or with threshold)

- Accepted run: run whose frequency is positive
- Accepted timed word: timed word with an accepted run

Examples



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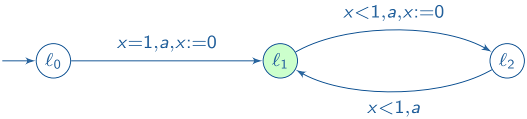

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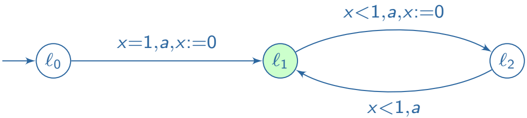

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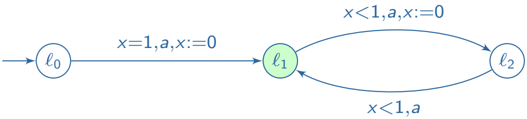

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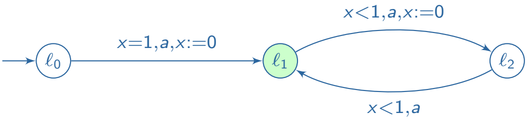

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Note that accumulated delays of run ρ_2 converge: it is a Zeno run.

Related works & Observations

Emptiness and Universality Problems in Timed Automata with Positive Frequency [BBBS11]

- restriction to one-clock timed automata
- non-Zeno: computation of the set of frequencies
- Zeno: computation of the bounds of the set of frequencies
  decide if they are reached

Optimal Infinite Scheduling for Multi-Priced Timed Automata [BBL08]

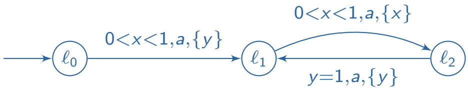
- restriction to strongly non-Zeno timed automata
- computation of an optimal run of an ε-optimal family

Goal: extension of the computation of the set of frequencies to n-clock timed automata.

Related works & Observations

Restriction to one-clock timed automata in [BBBS11]

- limitations due to convergency phenomena
- Ex:



Forgetfulness

- introduced in [BA11]
- a way to detect convergences

Assumptions: Forgetfulness + Strongly non-Zenoness

Corner-point abstraction [BBL08]

- Extension of the region abstraction
  - region + corner-point (*e.g.* region $x \in (0, 1)$, $\bullet$—)

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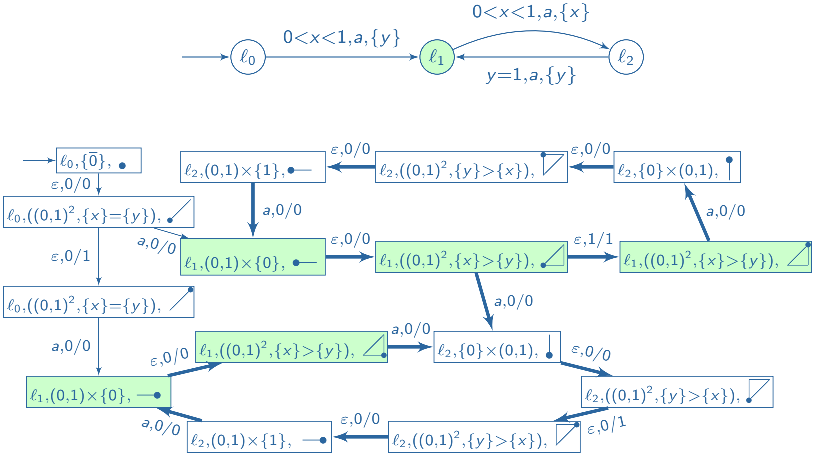
Lemma (reward-diverging case) [BBBS11]

$\text{Ratios(reward-diverging)} = \cup_{C \in scc}[m_C, M_C]$

where m_C: smallest ratio of a rew.-div. cycle in C
M_C: largest ratio of a rew.-div. cycle in C

Example


0<x<1,a,{x}
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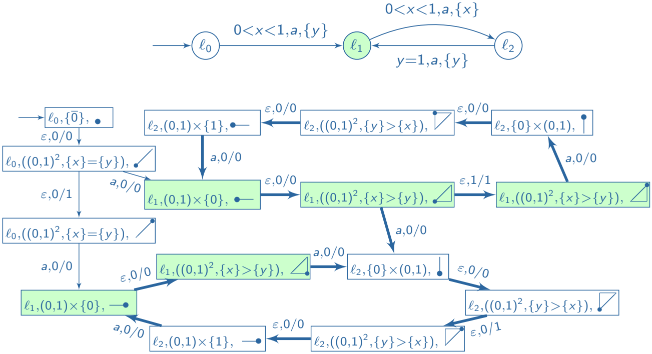
Forgetfulness of a cycle

A way to detect convergences:

- in [BA08]: orbit graph complete
- with the corner-point: projection strongly connected



Freqs($\mathcal{A}$) $= [0,1[$, Ratios($\mathcal{A}_{cp}$) $= \{0\} \cup \{1\}$

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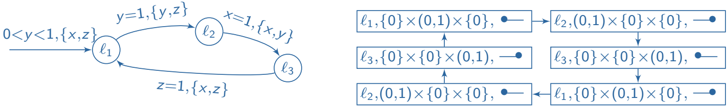

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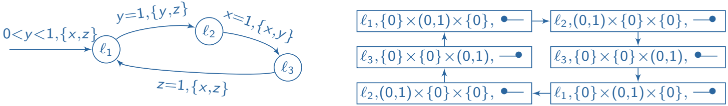

  - aperiodicity of a forgetful cycle: all the powers of the cycle are forgetful

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- $\rho_{\mathcal{A}}$ mimics $\pi_{\mathcal{A}_{cp}}$ up to ε if valuations of $\rho_{\mathcal{A}}$ have a distance to corners of $\pi_{\mathcal{A}_{cp}}$ smaller than ε (possible [BBL08])

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Goal

Goal: prove that $\mathrm{Freqs}(\mathcal{A}) = \mathrm{Ratios}(\mathcal{A}_{cp})(= \cup_{C \in SCC}[m_C, M_C])$

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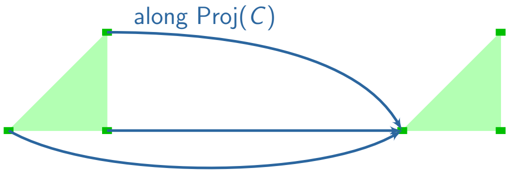
$\Rightarrow$ Let us show how forgetfulness can help.

Synchronizing cycle lemma



Lemma (Synchronization)

If there is a cycle $C : (\ell_0, r_0) \to \cdots \to (\ell_0, r_0)$ in $\mathcal{A}$ such that in the projection of C, there is a path from every corner α to α_0, then $\forall \varepsilon > 0$, from every $v \in r_0$, $\exists |v_{\alpha_0} - \alpha_0| < \varepsilon$ and a run along C from v to v_{α_0}.

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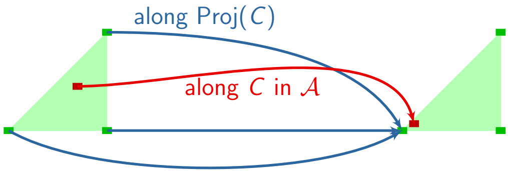

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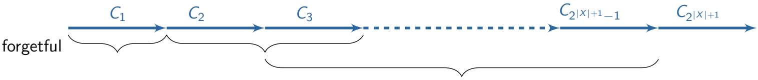

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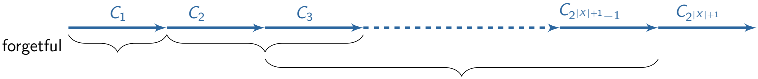

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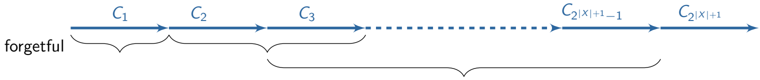

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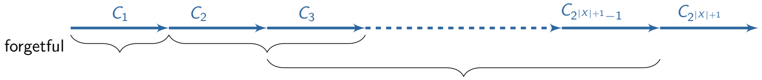

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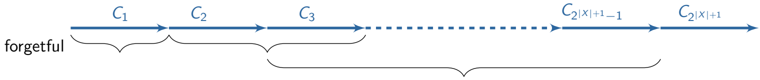

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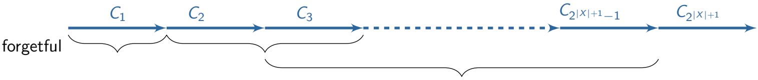

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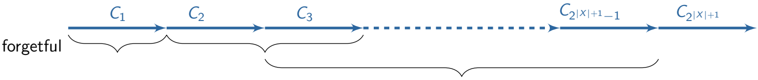

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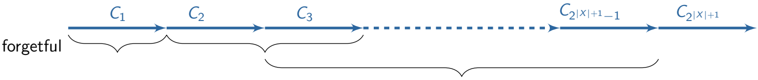

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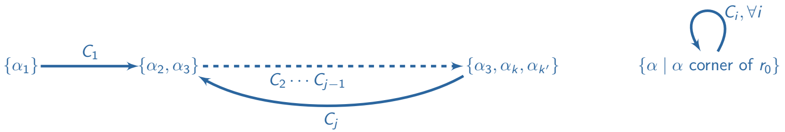

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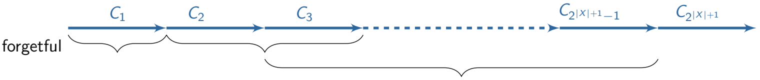

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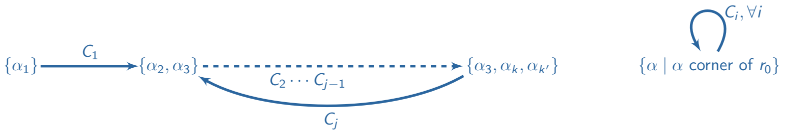

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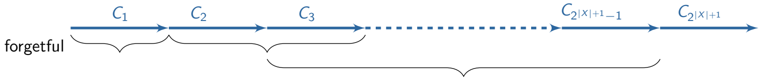

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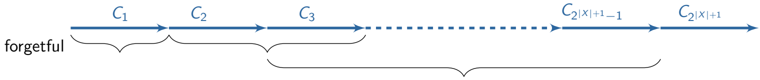

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Theorem

If $\mathcal{A}$ is a strongly non-Zeno, forgetful and aperiodic timed automaton, then

$$\mathsf{Freqs}(\mathcal{A}) = \mathsf{Ratios}(\mathcal{A}_{cp})$$

Conclusion

We extended the equality $\text{Freqs}(\mathcal{A}) = \text{Ratios}(\mathcal{A}_{cp})$ to n-clock timed automata under reasonable assumptions

- Not presented today:
  - one-clock:
    - cycle forgetful iff x is reset or not bounded
    - forgetful $\sim$ not "strongly Zeno"
    - forgetfulness $\not\Rightarrow$ strong non-Zenoness
    - we can compute the set of frequencies without strong non-Zenoness.
- Future work:
  - How synchronization lemmas could help in other context?
  - How forgetfulness could help?
    $\Rightarrow$ for discretization?